TRANSPORTATION PROBLEM
SOLUTION THROUGH ASSIGNMENT TECHNIQUE
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Compared to the Simplex method, Assignment and Transportation techniques are special purpose algorithms which are useful for solving some types of Linear Programming problems. The assignment problem itself is a special case of the Transportation problem. Usually for the initial allocation in the case of a Transportation problem methods such as North-west corner Rule, Vogel's approximation method, Least cost entry Method are used. For the purpose of optimality the MODI check is carried out finally. A specific condition is that the number of allocations should always equal m+n-1, where m=number of rows and n=number of columns.

An interesting feature of the Transportation problem arises in the case of initial allocation through the assignment technique. Consider a situation where the data constitute Selling Price, Cost of Production and Shipping costs. If the objective is to maximize it can be shown that each data can be construed separately as a transportation problem, i.e., Maximize Selling price, Minimize Cost of Production and Minimize Transportation Costs. If the Selling price and the Cost of Production are uniform to each column and row respectively, and the transportation costs is different it can be further shown that the solution obtained for the allocation of the Transportation cost individually will also be the solution for the Selling Price as well as Cost of Production, taken independently or collectively, ie., Selling Price minus Cost of Production. This is because of the fact that application of the assignment technique in case of Selling Price minus Cost of Production results in a constant difference and ultimately will lead to zero in the case of each cell. If all the cells are zero there are any number of solutions.

Consider the following problem.

A company has four factories situated in four different locations in the country and four sales agencies located in four other locations in the country. The cost of production (Rs. Per unit) the sales price (Rs. Per unit) shipping costs (Rs. Per unit) in the cells of matrix, monthly capacities and monthly requirements are given below:

	Sales Agency					
Factory	1	2	3	4	Maximum capacity	Cost of production (Rs.)
A	7	5	6	4	10	10
B	3	5	4	2	15	15
C	4	6	4	5	20	16
D	8	7	6	5	15	15
Monthly requirements	8	12	18	22		
Sale price Rs.	20	22	25	18		

Find the monthly production and distribution schedule, which will maximise profit.

In the above problem the usual procedure is to compute the profit for each of the sales agency corresponding to the factory. For example Factory A and Sales agency 1 the profit will be Rs.20-(10+7)=Rs.3. By applying Vogel's Approximation Method and MODI check the following optimal solution is obtained. It should be noticed that a minimum of 2 iterations is necessary to arrive at the optimal solution.

Factory	Sales Agency	Quantity in units	Profit per unit (Rs)	Total Profit (Rs)	Transportation cost per unit Rs.	Total Transportation cost Rs.
A	2	10	7	70	5	50
B	4	15	1	15	2	30
C	1	8	0	0	4	32
C	3	12	5	60	4	48
D	2	2	0	0	7	14
D	3	6	4	24	6	36
D	4	7	-2	-14	5	35
TOTAL		60		155		245

APPLICATION OF ASSIGNMENT TECHNIQUE.

As already stated the optimal allocation is found out only with respect to the Transportation cost. Applying the Assignment technique the following final cost matrix is obtained.

Initial matrix				Row operation				Column Operation			
7	5	6	4	3	1	2	0	3	0	2	0
3	5	4	2	1	3	2	0	1	2	2	0
4	6	4	5	0	2	0	1	0	1	0	1
8	7	6	5	3	2	1	0	3	1	1	0

After the column operation it can be noticed that element zero is found in six places. Out of which cells, R1C2, R2C4, R3C1, R3C3 and R4C4 can be allocated with the quantity of 10, 15, 8, 12 and 7 respectively. To satisfy the total condition of m+n-1, it is necessary to allocate in two more cells, with the next least cost. In this case the least cost is 1 which could be found in cells R4C2 and R4C3 which requires to be allocated. The balance quantity of 2 as well as 6 will be allocated in these cells. This solution will obviously be optimal since the allocation happens to be through the element zero and the next least cost 1, which is normally the basic property of any Assignment model. As already seen the solution is optimal, with the transportation cost being Rs.245.

Simple accounting equation suggests that the net profit equals Selling Price minus Cost. This aspect can be seen in the above problem. Since the selling price remains constant for each Sales Agency individually, the cost of production constant for each factory, the total sales equals Rs.1270 (Rs.20*8+22*12+25*18+18*22) and the cost of production equals Rs.870 (Rs.10*10+15*15+20*16+15*15). The transportation cost is Rs.245 and the net profit equals Rs.155, which was the solution obtained initially.

Application of assignment technique results in the elimination of a constant figure from each of the row as well as column. This could be used as a general case for all transportation problems. A major advantage is that the initial allocation can be easily carried out in the cells having zero as the element. If all the m+n-1 allocations is possible through zero element it can be easily concluded that such a solution is always optimal. However such situation is quite rare. In any case after allocating the resources in the zero cell, if the balance allocation is carried out in the next least cost apart from zero, one can still be towards the optimal solution. Almost all transportation problems whether balanced or unbalanced can be solved by using the Assignment method and in majority of the cases the solution is obtained in lesser number of iterations.