DEGENERACY AND TRANSPORTATION PROBLEM

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With respect to transportation problem for any non-degenerate solution the number of used squares must be equal to the number of rim requirements minus 1. Alternatively number of rows plus number of columns minus 1. Where this rule is not met the solution is Degenerate.

Failure to meet the test for degeneracy in the transportation problem is indicated in two ways:

1. There may be an excessive number of stone squares in a solution; the number of stone squares in greater than the number of rim requirements minus 1. This type of degeneracy arises only in developing the initial solution and is caused by an improper assignment or an error in formulating the problem. In such cases, one must modify the initial solution in order to get a solution which satisfies the rule of rim requirement minus 1.

2. There may an insufficient number of stone squares in a solution. Degeneracy of this type may occur either in the initial solution or in subsequent solutions. It is this type of degeneracy which requires special procedures to resolve. With an insufficient number of stone squares in a solution, it would be impossible to trace a closed path for each unused square, and with the MODI method it would be impossible to compute the row and column values.

In the recently concluded CA Final Examination, a problem on transportation (annexure) appeared in the paper Cost Management.

The above problem is a modified version of one of the classic problems on degeneracy. Further a peculiar aspect of the problem is that if a student starts with the North-west Corner Rule, for initial allocation it would clearly consume a minimum of $1\frac{1}{2}$ hours as there will be as many as 8 iterations. However under the Vogel's Method, the initial allocation itself is the optimal solution in-spite of it being a degenerate situation.

RESOLVING DEGENERACY:

To resolve degeneracy a zero allocation is assigned to one of the unused squares. Although there is a great deal of flexibility in choosing the unused square for the zero stone, the general procedure, when using the northwest corner rule, is to assign it to a square in such a way that it maintains an unbroken chain of stone squares. However Where the Vogel's Method is used the zero allocation is carried in a least cost independent cell. An independent cell in this context means that a cell which will not lead to a closed-path on such allocation.

With regard to the problem asked, the least cost cell happens to be R4C2 (cost Rs.3). But it is not independent. Since a closed path will be formed through the cells R1C2, R1C4 and R4C4. The next cell with least cost of Rs.5 is R4C3. This is independent as a closed path is not possible. With a zero allocation on this and with the MODI method for optimality the optimal cost is Rs.204/- (Solution in the web).

It can be seen that alternative solution also exist, but the number of allocation still remains at 7. Since what gets transferred from the cells is only zero. This leads to the following issues.

1. Whether a least cost independent cell will always be optimal
2. In case the problem is unbalanced whether the least cost independent cell, to be chosen can be either a cell in the row or a column with the least cost zero and
3. What happens if any cell which is independent is chosen?

The answer to the above is quite straight-forward. For issue 1, there is no necessity that a zero allocation in the least cost independent cell should always be optimal. It all depends upon the differences in the cost between the stone-squares. As regards issue 2, zero cost should be considered and in majority of the cases it can be seen that choosing a zero on such cells will lead to further iterations. The same is the case with respect to issue 3.

	C1	C2	C3	C4	C5	TOTA
R1	11	8 2	8	6 6	4 2	18
R2	10 9	9	12	9	6	10
R3	7	6	8 3	7	7	8
R4	2 9	3	5	2 6	11	4
TOTA	12	8	8	8	4	40

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