

## DEGENERACY AND SIMPLEX METHOD - R.SIVAKUMAR

Three grades of coal A, B and C contains phosphorus and ash as impurities. In a particular industrial process, fuel up to 100 tons (maximum) is required which could contain ash not more than 3% and phosphorus not more than 0.03%. It's desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of each grades are as follows:

Coal	Phosphorus (%)	Ash (%)	Profit in Rs. (per ton)
A	.02	3	12
B	.04	2	15
C	.03	5	14

You are required to formulate the Linear-programming (L.P.) model to solve it by using SIMPLEX METHOD to determine optimal product mix and profit. (Nov 05 CA Final)

### Critical observations of the problem

1. This problem involves the understanding of DEGENERACY in the case of Linear Programming problems. Degeneracy is a situation wherein one or more of the basic variables become Zero at some stage of the calculations. However in the above problem it can be noticed that "Zero" finds a place in the initial iteration itself. Further the coefficients of the variables were both positive and negative.
2. On preparation of the initial simplex table the ratios that are computed for replacing any one of the rows are found to be 0, 0 and 100. As per the study material /text books on the subject (including advanced text books, the ratio that is to be chosen is the one which is positive and also least among all the ratios. A ratio with Zero is not to be considered. Hence normally the ratio having the value 100 would have to be chosen. In that case the subsequent iteration would look like an optimal solution but it is not so since it does not satisfy the requirement of non-negativity i.e. variable S2 has a negative quantity. **(Refer solution)**.
3. If on the other hand if we proceed further by choosing any one of the other two ratios consisting of zero it can be noticed that iteration 1 and iteration 2 results in the same Z value i.e. Zero. At this point one would still be clueless since as per the simplex method every subsequent iteration should result in an improved Z Value. A situation of this kind clearly indicates the point of degeneracy. However if one were to go beyond the 2nd iteration one would find the optimal solution in the 4<sup>th</sup> iteration **(Refer Solution)**.

**Formulation:** Let  $x_1$ ,  $x_2$  and  $x_3$  be the tons of grade A, B and C coal respectively.

**Objective function - Maximize**  $Z=12X_1+15X_2+14X_3$

**Subject to:** 1.)  $-X_2+2X_3 \leq 0$ , 2.)  $-X_1+X_2 \leq 0$ , 3)  $X_1+X_2+X_3 \leq 100$  ( $X_1, X_2, X_3 \geq 0$ )

**Solution: Table 2**

Iteration 1		Qty							Ratio
Cj	Variable		X1	X2	X3	S1	S2	S3	
0	S1	0	0	-1	2	1	0	0	$0/-1 = -0$
0	S2	0	-1	1	0	0	1	0	$0/1 = 0$
0	S3	100	1	1	1	0	0	1	$100/1 = 100$
	zj	0	0	0	0	0	0	0	
	cj-zj		12	15	14	0	0	0	

**Iteration 2**

0	S1	100	1	0	3	1	0	1	
0	S2	-100	-2	0	-1	0	1	-1	
15	X2	100	1	1	1	0	0	1	
	Zj	1500	15	15	15	0	0	15	
	cj-zj		-3	0	-1	0	0	-15	

Note: Quantity of S2 is negative. Hence not Optimal .

**Iteration 2: Considering 0 as a ratio**

0	S1	0	-1	0	2	1	1	0	0
15	X2	0	-1	1	0	0	1	0	0
0	S3	100	2	0	1	0	-1	1	50
	zj	0	-15	15	0	0	15	0	
	cj-zj		27	0	14	0	-15	0	

**Iteration 3**

0	S1	50	0	0	2.5	1	0.5	0.5	40
15	X2	50	0	1	0.5	0	0.5	0.5	100
12	X1	50	1	0	0.5	0	-0.5	0.5	100
	zj	1350	12	15	13.5	0	1.5	13.5	
	cj-zj		0	0	0.5	0	-1.5	-13.5	

**Iteration 4 : Optimal**

14	X3	20	0	0	1	0.4	0.2	0.2	
15	X2	40	0	1	0	-0.2	0.4	0.4	
12	X1	40	1	0	0	-0.2	-0.6	0.4	
	zj	1360	12	15	14	0.2	1.6	13.6	
	cj-zj		0	0	0	-0.2	-1.6	-13.6	