

ASSIGNMENT MODEL – SOME INTERESTING PROPERTIES
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“The best person for the job” is an apt description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of workers to jobs, where any worker may undertake any job, albeit with varying degrees of skill. A job that happens to match a worker’s skill costs less than that in which the operator is not skillful. The objective of the model is to determine the optimum (least-cost) assignment of workers to jobs. The assignment problem is a special type of linear programming problem where assignees are being assigned to perform tasks. However, the assignees need not be people. They also could be machines, or vehicles, or plants, or even time slots to be assigned tasks.

PROPERTY I: Since an assignment model is always a square matrix it can be noticed that the final solution can always be synchronized in the diagonal. i.e., all the diagonal elements can be reduced to zero. This can be achieved by shifting the elements in a row or a column fully.

11	17	8	16
9	7	12	6
13	16	15	12
14	10	12	11

3	9	0	8
3	1	6	0
1	4	3	0
4	0	2	1

2	9	0	8
2	1	6	0
0	4	3	0
3	0	2	1

The solution is $8 + 6 + 13 + 10 = 37$. Now the elements in the above matrix can be shifted in such a way that diagonal elements become zero

13	16	15	12
14	10	12	11
11	17	8	16
9	7	12	6

1	4	3	0
4	0	2	1
3	9	0	8
3	1	6	0

0	4	3	0
3	0	2	1
2	9	0	8
2	1	6	0

PROPERTY 2: The solution any assignment problem equal the elements reduced from each row, each column and elements reduced in case the lines drawn does not equal the order of the matrix. In almost all the balanced situation, this property holds good. In the above example it can be seen that the elements from each of the row are $8+6+12+10$, and elements from the column 1, adds up to the solution.

PROPERTY 3: Where the elements of the matrix yields a constant difference it can be noticed that the minimum as well as the maximum are equal and one of the possible answers will lie at the diagonals. Example

3	7	11	15
19	23	27	31
35	39	43	47
51	55	59	63

Both the maximum and minimum equal 132.

PROPERTY 4 : If the assignment is on a proportion and if it is noticed that the elements are on a descending order the maximum occurs in the diagonal from North-west corner and the minimum occurs from North-east corner.

42	35	28	21
30	25	20	15
24	20	16	12
18	15	12	9

Maximum is $42+25+16+9 = 92$ and Minimum is $21+20+20+18 = 79$.

PROPERTY 5 : If all elements are different and if an element happens to be the minimum of the row as well as the column. It will enter the solution invariably. This property could be used in a restricted sense to reduce the order of the matrix. Where the order of the matrix is reduced to two the final answer is the diagonal elements of the 2×2 matrix plus the elements already noticed. In the above example (in property 1) 8 and 6 happens to be the minimum in row 1 and column 3, row 2 and column 4. After their elimination the net 2×2 matrix will be 13,16 and 14,10. Hence the least of the diagonal elements i.e, $13+10$ or $16+14$ will have to be chosen. Obviously $13+10$ is the answer. Final solution will be $13+10+6+8=37$.